ALTA
Institute for Algebra, Geometry, Topology and their Applications

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Discrete Mathematics
WS 2025/26

Bonus Exercise Set

Exercise B.1 Show that for any nonnegative integer t, we have the identity

$$\binom{8t}{0} - \binom{8t}{2} + \binom{8t}{4} - \cdots + \binom{8t}{8t} = 16^t.$$

(6 Punkte)

Exercise B.2 Let n be a positive integer.

(1) Assume t is a nonnegative integer, such that $n \geq 2t+1$, show that

$$S(n, 2t+1) \equiv \binom{n-t-1}{t} \mod 2.$$

(2) Assume t is a positive integer, such that $n \geq 2t$, show that

$$S(n, 2t) \equiv \binom{n-t-1}{t-1} \mod 2.$$

(6 Punkte)

Exercise B.3 The numbers $\{P_n\}_{n\geq 0}$ are recursively defined by

$$P_0 = 0, \ P_1 = 1,$$
$$P_{n+2} = 2P_{n+1} + P_n, \text{ for } n \geq 0.$$

Show the identity

$$P_{n+1}P_{n-1} = P_n^2 + (-1)^n,$$

for all $n \geq 1$.

(6 Punkte)

Exercise B.4 Assume G is a directed graph with at least two vertices, such that for any $v, w \in V(G)$, exactly one of (v, w) and (w, v) is in $E(G)$. Show that if any vertex in G can be reached from any other vertex by a directed path, then G has a directed Hamiltonian cycle.

(6 Punkte)

Submission of the bonus exercises: Tues, 6.1.26, before the tutorial (until 12:15) into the postbox 54 in MZH 1st floor, or submission at the beginning of the 12:30-tutorial.