ALTA
Institute for Algebra, Geometry, Topology and their Applications

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Discrete Mathematics
WS 2025/26

Exercise Set 3

Exercise 3.1 Let a and b be positive integers. Find the number of strings s consisting 0's and 1's such that

(1) the number of 0's is a and the number of 1's is b,
(2) the string s does not contain two consecutive 0's.

For instance, for $a = b = 2$, there are 3 allowed strings: 0101, 0110, and 1010.

(6 Punkte)

Exercise 3.2 Assume $n \geq 3$, find the formulas for:

(1) $S(n, 3)$,
(2) $S(n, n-2)$.

(6 Punkte)

Exercise 3.3 Given a positive integer n and a prime number p, give a formula for the number of those $0 \leq k \leq n$, for which $\binom{n}{k}$ is not divisible by p, and prove it.

(6 Punkte)

Exercise 3.4 A nonnegative integer is called fibbinary if its binary presentation does not contain two consecutive 1's. The first fibbinary numbers are $0, 1, 2, 4, 5, 8, 9, 10, 16, 17, 18, \ldots$. Show that n is fibbinary if and only if the binomial coefficient $\binom{3n}{n}$ is an odd number. (6 Punkte)

Submission of the exercises: Tues, 04.11.25, before the tutorial (until 12:15) into the postbox 54 in MZH 1st floor, or submission at the beginning of the 12:30-tutorial.