**ALTA**
**Institute for Algebra, Geometry, Topology and their Applications**

**Prof. Dr. Dmitry Feichtner-Kozlov**
**Discrete Mathematics**
**WS 2025/26**

## Exercise Set 9

**Exercise 9.1** Give an example of a network $N$, with a flow $f$, such that the flow is not maximum, and there is no way to improve the flow by increasing along a path using only forward pointing edges. (In other words, using augmented paths in the algorithm is really essential!)

(6 Punkte)

**Exercise 9.2** Is it possible to find a network with maximal flows $f$ and $f'$, such that the values of $f$ and $f'$ are different on all the edges?

(6 Punkte)

**Exercise 9.3** Let $N$ be a network with the source $s$ and the sink $t$. An *elementary flow* is a flow which is obtained by assigning a constant positive value to the set of edges on a directed path from $s$ to $t$, and assigning value 0 to all other edges.

Show that every flow is the sum of elementary flows.

(6 Punkte)

**Exercise 9.4** Let $N$ be a network, and let $(S, T)$ and $(S', T')$ be cuts of minimum capacity. Prove that the cut $(S \cup S', T \cap T')$ also has a minimum capacity.

(6 Punkte)

**Submission of the exercises:** Tues, 13.01.26, before the tutorial (until 12:15) into the postbox 54 in MZH 1st floor, or submission at the beginning of the 12:30-tutorial.